$$\frac{d^2y}{dx^2}=\left\{\frac{a^2\dfrac{d\rho}{da}}{\displaystyle\int_0^a \rho a^2 da}+\frac{6}{a^2}\right\}y \quad\ldots\ldots\ldots\ldots\ldots\ldots\ldots(2).$$

It is due to this connection, that the proposed equation has received a variety of solutions.

What is here given is based on a transformation of (1) and a pair of solutions given in Johnson's *Differential Equations*, articles 213, 214.

(1) is transformed into

$$\frac{d^2u}{dx^2}-\frac{m-1}{x}\frac{du}{dx}-a^2u=0 \quad\ldots\ldots\ldots\ldots\ldots\ldots\ldots(3),$$

two integrals of which are of the form

$$u_3=e^{ax}\left(1-\frac{m-1}{m-1}ax+\frac{(m-1)(m-3)}{(m-1)(m-2)}\frac{a^2x^2}{2\,!}-\ldots\right) \quad\ldots\ldots\ldots\ldots(4),$$

$$u_5=e^{-ax}\left(1+\frac{m-1}{m-1}ax+\frac{(m-1)(m-3)}{(m-1)(m-2)}\frac{a^2x^2}{2\,!}+\ldots\right) \quad\ldots\ldots\ldots\ldots(5),$$

and connected with u by the relation $u=Au_3+Bu_5$ ....(6).

Now assuming $u=x^pv$.........(7), or $v=ux^{-p}$.........(8), (3) becomes

$$\frac{d^2v}{dx^2}-a^2v=\frac{p(p+1)v}{x^2} \quad\ldots\ldots\ldots\ldots\ldots\ldots\ldots(9).$$

Putting $a=n\surd(-1)$, $p=2$, $m=2p+1=5$, and $v=y$, $\surd(-1)=i$, (6) and (8), with (4) and (5) give

$$y=ux^{-2}=x^{-2}[Ae^{nix}+Be^{-nix}-\tfrac{1}{3}n^2x^2(Ae^{nix}+Be^{-nix})+nx(Bie^{-nix}-Aie^{nix})] \quad\ldots(10),$$

which it is not difficult to reduce to the form,

$$y=Cx^{-2}[(3-n^2x^2)\cos(nx+\alpha)+3nx\sin(nx+\alpha)] \quad\ldots\ldots\ldots\ldots(11).$$

See Forsyth's *Differential Equations*, Ed. 1885, Ex. 3, page 65; Ex. 1, page 175; Ex. 21, page 180; Ex. 26, page 181, and Ex. 5, pages 233, 4.

76. Proposed by E. B. ESCOTT, Cambridge, Mass.

Solve the partial differential equation, $q^2r+4pqs+p^2t+p^2q^2(rt-s^2)=a^2$. [*Forsyth's Differential Equations*, page 376.]

Solution by G. B. M. ZERR, A. M., Ph. D., Professor of Mathematics and Science, Chester High School, Chester, Pa.

The equation in λ is, $\lambda^2p^2q^2(1+a^2)+4\lambda p^3q^3+p^4q^4=0$.

$\therefore\ \lambda^2(1+a^2)+4\lambda pq+p^2q^2=0$.

$\therefore\ \lambda=-\{pq/(1+a^2)\}[1\mp\surd(3-a^2)]=m_1pq$ or m_2pq.

The first system of integrals is $p^2q^2dy+m_1p^3qdx+m_1p^3q^3dp=0$.

$p^2q^2dx+m_2pq^3dy+m_2p^3q^3dq=0$, or $qdy+m_1pdx+m_1pq^2dp=0$...........(1);

$pdx+m_2qdy+m_2p^2qdq=0$ ....(2).

Similarly for second system $pdx+m_1qdy+m_1p^2qdq=0$............(3);

$qdy+m_2pdx+m_2pq^2dp=0$ ....(4).

(1)+(3) gives $(pdx+qdy)(1+m_1)+m_1(pq^2dp+p^2qdq)=0$, but $dz=pdx+qdy$.

$\therefore$ $(1+m_1)dz+m_1)pq^2dp+p^2qdq)=0$.

$\therefore$ $2z(1+m_1)+m_1p^2q^2=2a$ ....(5).

Similarly (2)+(3) gives $2z(1+m_2)+m_2p^2q^2=2b$.................(6).

Eliminating p^2q^2 between (5) and (6) we get $z(m_1-m_2)=bm_1-am_2$.

77. Proposed by T. E. COLE, Columbus, Ohio.

Derive the equation of a point in a pedal of a bicycle as the wheel rolls along on a plane.

I. Solution by G. B. M. ZERR, A. M., Ph. D., Professor of Mathematics and Science, Chester High School, Chester, Pa.; and A. E. BREECE, A. B., Professor of Mathematics, Portland University, Portland, Oregon.

Let CO be the radius of a circle whose circumference is equal in length to the distance C moves in one revolution of the pedal. P a point in the pedal, A, A' points where CP is perpendicular to the fixed line AA'.

Let $CO=a$, $CP=d$, $\angle OCP=\theta$, then we have

$x=AN=AO-ON=a\theta-d\sin\theta$; $y=PN=CO+PM=a-d\cos\theta$.

$\therefore$ $x=a\cos^{-1}[(a-y)/d]-\sqrt{[d^2-(a-y)^2]}$.

If $a=d$, the curve is a right cycloid.

If $a<d$, the curve is a prolate cycloid.

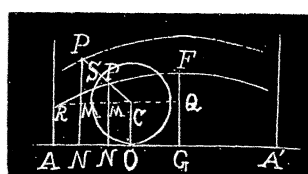
If $a>d$, the curve is a curtate cycloid.

II. Solution by WALTER HUGH DRANE, A. M., Professor of Mathematics, Jefferson Military College, Washington, Miss.

Consider the curve traced by the pedal in one turn. Take origin at the lowest point of the pedal's path. Let axis of y be vertical and axis of x the straight line joining any two consecutive lowest points of pedal's path. Take P any point in the curve and draw its ordinate PN. Let F be the middle point of curve and FG its ordinate. From Q, the middle point of FG, draw QR parallel to the x-axis. Take $PC=FQ$ and draw CO parallel to PN.

Now let a be length of pedal arm, r the radius of bicycle wheel, n the number of turns the wheel makes to one turn of the pedal, and θ the angle PCO through which the pedal arm has turned on reaching the point P in the curve.

Then from the figure, we have,



$$x=AN=AG-(GO+MC)=\pi rn-\left[\left(\pi rn-\frac{\pi rn\theta}{180}\right)+a\sin\theta\right]=\frac{\pi rn}{180}\theta-a\sin\theta.$$

$$y=PN=MN+MP=a-a\cos\theta=a\operatorname{vers}\theta.$$

Whence the equation of curve is, $x=\dfrac{\pi rn}{180}\operatorname{vers}^{-1}\dfrac{y}{a}-\sqrt{(2ay-y^2)}$.